Transformer Leakage Flux Models for Electromagnetic Transients: Critical Review and Validation of a New Model

Mathieu Lambert, *Member, IEEE*, Manuel Martínez-Duró, *Member, IEEE*, Jean Mahseredjian, *Fellow, IEEE*, Francisco de León, *Senior Member, IEEE*, and Frédéric Sirois, *Senior Member, IEEE*

Abstract—This paper presents experimental validation of the coupled leakage inductance transformer model. It is shown that the coupled approach yields the same results as the indefinite admittance matrix method of BCTRAN. A topologically correct three-phase shell-type transformer model is proposed. The connection points between the leakage and magnetizing inductances are properly identified, which makes the new model superior to BCTRAN and the hybrid models by providing physical consistency. In addition, experimental verification of a method to calculate the short-circuit inductances is presented. New explanations on the division of leakage flux and on the mathematical equivalence between the T- and Π-equivalent models are also given.

Index Terms—Leakage inductance, low-frequency electromagnetic transients, topological model, transformer modeling.

Nomenclature

N	Number of windings (or coils, if the windings are subdivided).
N_c	Number of common turns. Can be set to one.
N_1, N_2	Number of turns for the primary and secondary windings, respectively.
ϕ_l	Leakage flux.
ϕ_c	Core flux.
$\mathscr{R}_l$	Leakage reluctance.
$\mathscr{R}_c$	Core reluctance.
$\mathscr{F}$	Magnetomotive force.
$\mathscr{L}$	Coupled leakage inductance.
λ	Flux linkage.

i	Current.
L_l	Leakage inductance.
L_c	Core inductance.
L_{c_sat}	Saturated core inductance.
L_sc	Short-circuit inductance.
l_c	Core length.
l_tot	Total core length.
A_c	Core cross section.
μ	Permeability.
K	Leakage inductance split ratio.

I. Introduction

THE MOST accurate transformer models for low-frequency electromagnetic transients (below the winding first resonance frequency, typically a few kilohertz ([1], Section 1)) have a physical basis. In these models, the magnetic flux is confined inside predefined paths called flux tubes. These models are called *topological*, since each model element represents a part of the reluctance in the magnetic-field physical path. These models are used in electromagnetic-transients (EMT)-type programs instead of vectorial field models, because the computational cost involved with FEM simulations would be prohibitive due to three facts: 1) the transient nature of the phenomenon, which would require computing a field solution for each time-step; 2) the nonlinearities of transformer cores; and 3) the need to model not just one transformer but several of them (depending on the system configuration being studied).

Even though the more recent models proposed in the literature are topological and, thus, physically based, it can be seen that for a given transformer configuration, many different "topological" models exist. For instance, for the three-phase three-legged stacked-core transformer (with two and three windings), a shortened list of models is given by [2, Fig. 5.5], [3, Fig. 4.18], [4, Fig. 3], [5, Fig. 2.15], [6, Fig. 2], [7, Fig. 6(b)], and [8, Fig. 2]. If the models are physically based, why are there so many different topological models for the same transformer type? The answer is: because of leakage flux.

In theory, if magnetic insulators existed, the magnetic flux could be entirely confined within the magnetic "conductors," as is the case for electric circuits at low frequencies. The key difference between electric and magnetic circuits is the difference in the relative values of magnetic permeability and conductivity between conductors and insulators. Since the ratio of permeabilities of oil (or air) and iron is in the range of 10^3–10^4, some of the magnetic flux leaks from the magnetic core, whereas the ratio of conductivities of pure air (or polymers) and copper is in the order of 10^{22}–10^{28}. According to [9], finding an equivalent magnetic circuit with leakage flux is similar to deriving an equivalent electric circuit for a network of copper conductors immersed in an electrolyte solution. Hence, the chosen paths (or flux tubes, to be more precise) and the points of connection of leaks to the circuit can vary from one model to another.

Despite the fact that discrepancies exist due to the particular choice of flux paths, another key difference comes from the fact that some models *divide* the leakage flux between a pair of windings into two fluxes, while others do not. This can be seen in [10, Figs. 7 and 8], where the leakage flux is divided, compared to [6, Figs. 1 and 2], where it is not divided. This difference in the modeling of leakage flux is highlighted in [5, Chs. 1 and 2], where the two approaches were, respectively, called the *integral flux* approach and *divided flux* approach. Both approaches are explained in Section II.

The main problem with those topological models is their failure to adequately represent short-circuit conditions for transformers with more than two windings (see [11, Sec. III]). For topological models of N-winding transformers based on the integral flux approach, there are $N - 1$ leakage inductances, and for those based on the divided flux approach, there are N leakage inductances. Since there are $N(N - 1)/2$ different pairs of short-circuit combinations, it can be seen that these approaches will fail to represent all cases (they can work only for a given number of windings N).

To alleviate this problem, it was proposed in [4], [12], and [13] to use coupled inductors (or a mesh-equivalent network) to reproduce the $N(N - 1)/2$ pairs of short-circuit conditions, where the indefinite admittance matrix can be calculated with the method proposed in [14] (BCTRAN). However, this representation of leakage inductances is no longer topological, it is just a solution for reproducing more accurately the terminal short-circuit measurements. This is highlighted in [13, p. 77]. In order to connect a topological core model to the short-circuit admittance matrix, one or more fictitious windings are necessary, where a portion of the short-circuit admittance is divided with this approach. This approach was called the *hybrid model* in [4] and [15] to emphasize the fact that it uses a topological core model with a more appropriate representation of short-circuit admittances.

Even though the hybrid approach solves the problem of inadequacy between terminal short-circuit measurements and the values calculated with topological leakage models, it raises other issues. First, how many fictitious windings are required? According to [12] and [16], there are two fictitious windings α and β, whereas there is only one in [4]. Second, how should the short-circuit admittance be split to account for these windings? The proportionality factor K of [4], to derive a "leakage induc-

tance" between the inner winding and the fictitious winding, is proposed to be $K = 0.5$ in [17]. In [6], a value of $K = 0.33$ is calculated from classical ampere-turn diagram formulas, assuming that the fictitious winding is infinitely thin. However, it was shown in [18] that these formulas can be inaccurate if the flux is not axial. Furthermore, in [12], a coupling factor $\delta = 10^3$ is used to couple the short-circuit admittance matrix to the fictitious winding. Third, it is mentioned in [17] that: "*The artificial core winding is related to the leakage channel between the inner physical winding and the core.*" However, which winding is the inner winding in the case of a sandwiched winding design? The issues raised here show the physical inconsistencies that exist with this approach.

Based on the aforementioned arguments, it can be concluded that a topological leakage model was needed. Such a model must reproduce the terminal short-circuit measurements without the use of fictitious windings.

This type of model was first presented in [11], but the objective of this paper was to solve the numerical instability problems caused by the negative inductance in the star-equivalent circuit of a three-winding transformer, with the use of coupled leakage inductances. This principle was generalized to single-phase N-coil[1] transformers in [19].

In this paper, it is first demonstrated how this model is related to the integral flux approach and it is also explained why this approach is better. Afterwards, the mathematical equivalence between the divided and integral flux approaches is shown for a single-phase two-winding transformer, assuming the core inductances are linear. Then, a new relationship is derived for the well-known T-network model (divided flux approach) to calculate the correct ratio on how to split the short-circuit inductance between the two leakage inductances, according to the core and windings' geometries. It is also demonstrated that when the core is considered nonlinear, for the T-network model to be mathematically equivalent, it requires nonlinear leakage inductances. How to calculate the coupled leakage inductance matrix from the short-circuit inductance matrix is explained next. Finally, this paper also presents a new model for a three-phase shell-type transformer that uses the coupled leakage approach.

The main contributions of this paper are the validation with measurements of the coupled leakage inductance model, developed in [19], and the method presented in [18] to calculate its short-circuit inductances; new explanations on the division of leakage flux and on the mathematical equivalence between the T- and Π-equivalent models; and the extension of the method presented in [19] to a three-phase shell-type transformer model.

II. INTEGRAL AND DIVIDED FLUXES

In order to understand the differences between the two methods, let us look at a simple example: the single-phase two-winding shell-type transformer with cylindrical windings. In the divided flux approach, the magnetic flux is split into three components: 1) the leakage flux of winding 1 ϕ_{l_1}; 2) the leakage flux of winding 2 ϕ_{l_2}; and 3) the common (or main) flux ϕ_c, as shown in Fig. 1(a). For the integral flux approach, this division is not made and only one leakage flux can exist,

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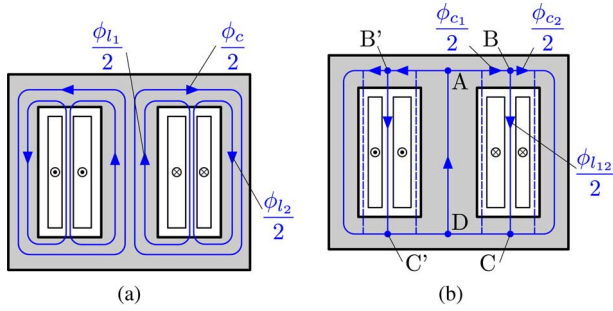[1]*Coil* refers to a fraction of a winding.

Fig. 1. Magnetic flux paths of a single-phase two-winding shell-type transformer. (a) Divided flux approach. (b) Integral flux approach.
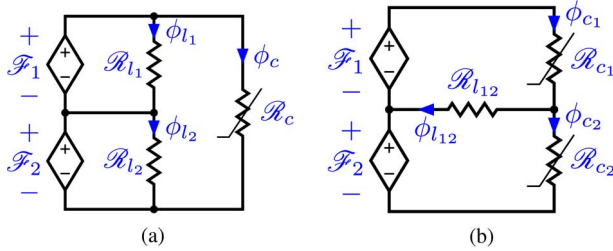

Fig. 2. Equivalent magnetic circuits of a single-phase two-winding shell-type transformer. (a) Divided flux approach. (b) Integral flux approach.
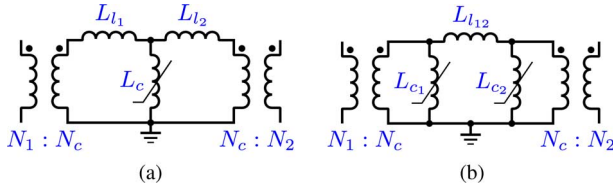

Fig. 3. Dual electric circuits of a single-phase two-winding shell-type transformer. (a) Divided flux approach. (b) Integral flux approach.

namely, the flux $\phi_{l_{12}}$ circulating between the windings, as seen in Fig. 1(b).[2] The equivalent magnetic circuits (after simplifications[3]) of both approaches are illustrated in Fig. 2, and the dual electric circuits are shown in Fig. 3. It can be seen that for this example, the divided flux approach leads to the well-known T-network, whereas the integral flux approach results into the Π-network.[4] Even though the two approaches are related through the star-delta transformation (if the nonlinear elements are assumed linear) and are equivalent for this example, it is not generally the case for all integral flux and divided flux magnetic circuits (e.g., with more than two windings). Thus, it is necessary to investigate which method is more appropriate and more generic, to model a transformer on a topological (physical) basis.

[2]The shunt air paths (shown with dashed lines) are usually omitted, since they can be combined with the core's nonlinear reluctances $\mathscr{R}_{c_1}$ and $\mathscr{R}_{c_2}$.

[3]Since the geometry is symmetric along the axis AD, both halves are in parallel (B and B′ are virtually connected and so are C and C′, because they have the same magnetic scalar potentials). Therefore, ϕ_{c_1} is the flux flowing in the path CC′-D-A-BB′, ϕ_{c_2} is the flux flowing in the core from BB′ to CC′, and $\phi_{l_{12}}$ is the flux through the air from BB′ to CC′.

[4]For the core-type transformer, the same procedure can be followed and the dual circuit for the integral flux approach will be the same, as illustrated in [20, Fig. 4.8].

This question was studied in [21]–[23] and later recalled in [5]. It was concluded that the T-network, hence the divided flux approach, is merely the result of mathematical manipulations, whereas the Π-network (integral flux approach) is physically based. Furthermore, it is highlighted in [5] that the star-delta transformation is only valid for linear elements. Thus, the integral flux approach should be privileged over the divided flux approach for the derivation of a topological transformer model, since the mathematical equivalence (existence and uniqueness of a solution) between the models is no longer guaranteed.

Despite the fact that the model mostly used the T-network for single-phase two-winding transformer, a few authors also proposed the Π-network (see, for instance, [24]–[26, pp. 250–251]. More recently, this model was also covered in ([20, Sec. 4.4.2] and in [27], where it is mentioned that the Π-network is the topological model for the single-phase transformer.

The division factor between the leakage inductances L_{l_1} and L_{l_2} of the T-network and the short-circuit inductance, which is typically assumed to be 0.5 or 0.75–0.9 [28, p. 2053], actually depends on the division of the core inductances of the Π-network (ratio of core lengths). This is demonstrated in the next section, using the star-delta transformation.

III. STAR-DELTA TRANSFORMATION

Even though it was claimed that the star-delta transformation cannot be used for nonlinear elements in [5], it is useful to demonstrate the relationship that exists between the integral flux and the divided flux approaches for single-phase transformers from a piecewise linear perspective. The inductances L_{l_1}, L_{l_2} and L_c in the divided flux approach are related to the inductances in the integral flux approach through the star-delta transformation

$$L_{l_1} = \frac{L_{c_1} L_{l_{12}}}{L_{c_1} + L_{c_2} + L_{l_{12}}} = K_1 L_{l_{12}} \tag{1}$$

$$L_{l_2} = \frac{L_{c_2} L_{l_{12}}}{L_{c_1} + L_{c_2} + L_{l_{12}}} = K_2 L_{l_{12}} \tag{2}$$

$$L_c = \frac{L_{c_1} L_{c_2}}{L_{c_1} + L_{c_2} + L_{l_{12}}}. \tag{3}$$

Since $L_{c_1} + L_{c_2} \gg L_{l_{12}}$ if the core is unsaturated, it can be approximated that

$$L_c \approx (L_{c_1} L_{c_2})/(L_{c_1} + L_{c_2}) \tag{4}$$

$$K_1 \approx L_{c_1}/(L_{c_1} + L_{c_2}) \tag{5}$$

$$K_2 \approx L_{c_2}/(L_{c_1} + L_{c_2}) \tag{6}$$

so that $K_1 + K_2 \approx 1$ and L_c approximately equal to L_{c_1} in parallel with L_{c_2}. Considering from Fig. 3 that $N_c = 1$, the inductances can be rewritten in terms of the permeability μ, the core lengths l_{c_1}, l_{c_2} and the core cross-sections A_{c_1}, A_{c_2}

$$K_1 \approx \frac{\frac{\mu A_{c_1}}{l_{c_1}}}{\frac{\mu A_{c_1}}{l_{c_1}} + \frac{\mu A_{c_2}}{l_{c_2}}} \tag{7}$$

$$K_2 \approx \frac{\frac{\mu A_{c_2}}{l_{c_2}}}{\frac{\mu A_{c_1}}{l_{c_1}} + \frac{\mu A_{c_2}}{l_{c_2}}}. \tag{8}$$
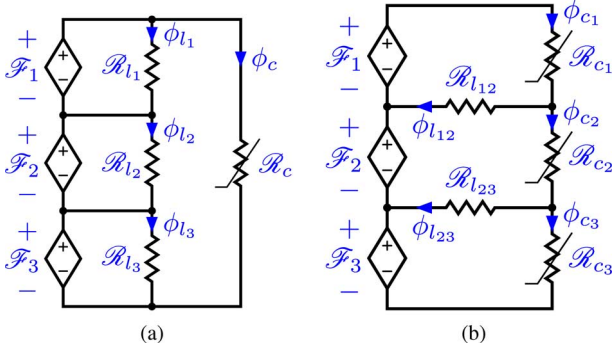
Fig. 4. Equivalent magnetic circuits for the three-winding single-phase shell-type transformer. (a) Divided flux approach. (b) Integral flux approach.
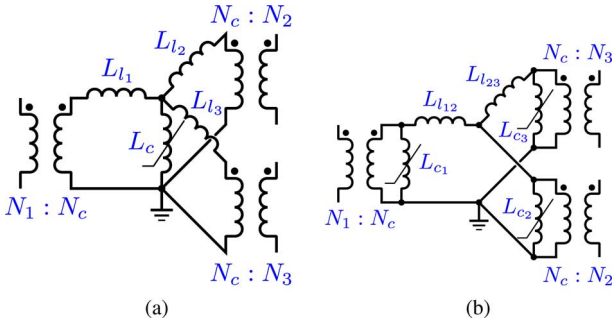

Fig. 5. Dual electric circuits for the three-winding single-phase shell-type transformer. (a) Divided flux approach. (b) Integral flux approach.

In the case of a core-type single-phase transformer, $A_{c_1} = A_{c_2}$. For a shell-type single-phase transformer, this is also the case if the outer leg cross-section is half the cross-section of the center leg.[5] Hence, in both cases, (7) and (8) become

$$K_1 \approx l_{c_2}/l_{\text{tot}} \qquad (9)$$

$$K_2 \approx l_{c_1}/l_{\text{tot}} \qquad (10)$$

where $l_{\text{tot}} = l_{c_1} + l_{c_2}$ is the total length.

An important conclusion is that in the linear case, the primary leakage inductance L_{l_1} is proportional to the ratio l_{c_2}/l_{tot} and the secondary leakage inductance L_{l_2} is proportional to the ratio l_{c_1}/l_{tot}. This claim was also made in [29, p.71], but without explanation. It is also mentioned that the two models are mathematically equivalent, which is true with linear (unsaturated) inductances.

If we consider that L_{c_1} and L_{c_2} are represented by the two-slope piecewise linear curve presented in Fig. 6, in the case where the core is fully saturated we obtain

$$L_{l_1} = \frac{L_{c_1\text{sat}} L_{l_{12}}}{L_{c_1\text{sat}} + L_{c_2\text{sat}} + L_{l_{12}}} = K_1 L_{l_{12}} \qquad (11)$$

$$L_{l_2} = \frac{L_{c_2\text{sat}} L_{l_{12}}}{L_{c_1\text{sat}} + L_{c_2\text{sat}} + L_{l_{12}}} = K_2 L_{l_{12}} \qquad (12)$$

$$L_c = \frac{L_{c_1\text{sat}} L_{c_2\text{sat}}}{L_{c_1\text{sat}} + L_{c_2\text{sat}} + L_{l_{12}}} \qquad (13)$$

[5]Note that in practice, for shell-type power transformers, the outer legs (and yokes) cross-sections may be larger than half the cross-section of the center leg. In that case, the reluctance $\mathscr{R}_{c_1}$ that comes from the flux path CC′-D-A-BB′ (as mentioned earlier) would need to be divided into two reluctances (in series). One for the center leg (path D-A) and one for the two yokes (paths CC′-D and A-BB′).
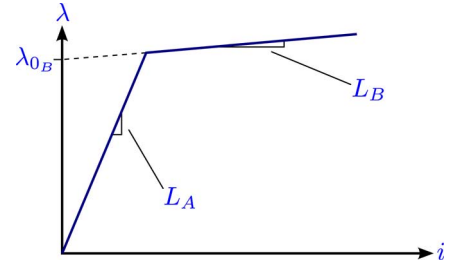

Fig. 6. Two-slope piecewise-linear magnetizing curve for L_{c_1} and L_{c_2} of the Π equivalent.
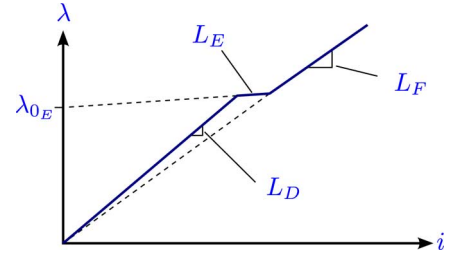

Fig. 7. Resulting piecewise-linear leakage curve for L_{l_1} and L_{l_2} of the T equivalent.

where the saturation inductances $L_{c_1\text{sat}}$ and $L_{c_2\text{sat}}$ (represented by L_B in Fig. 6) are now in the order of magnitude of $L_{l_{12}}$ (since they all represent flux tubes with air permeability μ_0). Hence, it means that the leakage inductance split ratios K_1 and K_2 are not constants but functions of the saturation level of the core. Furthermore, if L_{c_1} starts to saturate, but L_{c_2} is not yet saturated, we have the intermediate case where $K_2 \approx 1$, and conversely, when L_{c_2} starts to saturate, but L_{c_1} is not yet saturated (again assuming two-slope saturation curves), then $K_1 \approx 1$. Note that the core inductance L_c in the divided flux approach is also dependent on the leakage inductance $L_{l_{12}}$, as the core starts to saturate.

The resulting curves for L_{l_1} and L_{l_2} are shown in Fig. 7 and for L_c in Fig. 8, for this particular case. An example with numerical values is presented in the Appendix. Thus, the two models are mathematically equivalent only if we consider nonlinear partial (divided) leakages for the T model. This partially explains the difference observed between both models in [30], where the divided leakage inductances were considered linear. In that case, if the parameters are made to fit measurements on one side for the T-equivalent model (with linear leakage inductances), the behavior of this model will be incorrect viewed from the other side. Hence, the T model will not be reversible[6] with linear leakage inductances. However, the reversibility of the Π-equivalent model was demonstrated in [27] and later in [31].

Physically, it makes sense to have nonlinear divided leakages, because the divided leakage fluxes partially link the core, as shown in Fig. 1(a). Furthermore, in the T-equivalent model theory, leakage inductances are defined as the difference between self and mutual inductances [9, p. 314], where both are

[6]By "not reversible," it is meant that if the parameters are calculated to fit with the nonlinear curve seen from one terminal, the nonlinear behavior will be incorrect seen from the other terminal.
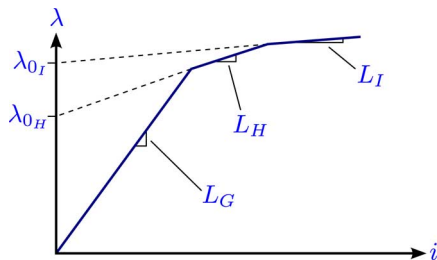
Fig. 8. Resulting piecewise-linear magnetizing curve for L_c of the T equivalent.
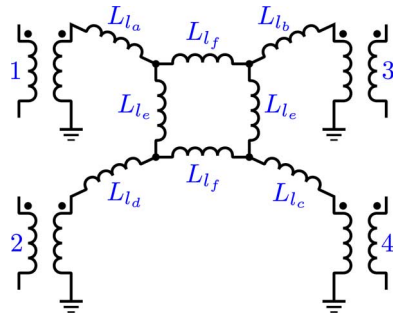

Fig. 9. Star equivalent circuit for the single-phase four-winding transformer (core inductances not shown).
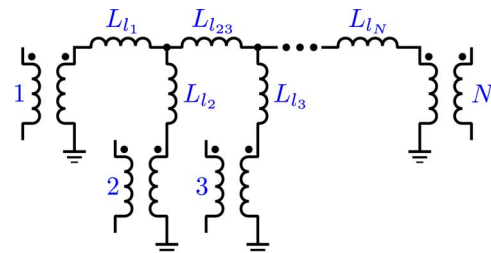

Fig. 10. Simplified star equivalent circuit for the single-hase N-winding transformer (core inductances not shown).

nonlinear. Since the correct representation of leakage in the divided flux approach is more complex than in the integral flux approach, it is preferable to use the later representation. The generalization to a N-winding transformer is also easier that way, as shown in the next section.

IV. COUPLED LEAKAGE APPROACH

As mentioned in Section I, the problem with most topological models is their inability to reproduce the short-circuit measurements for more than two windings for the integral flux approach, and for more than three windings for the divided flux approach. This is explained in the following paragraphs.

By adding a third cylindrical winding to the transformer, the equivalent magnetic circuit of Fig. 2(a) becomes the one in Fig. 4(a), and its dual electric circuit is shown in Fig. 5(a), which is commonly referred to as the *star circuit* [32]. The fact that the star circuit is not valid for more than three windings in [32, p. 14] can be observed from Fig. 5(a), where a fourth winding would add a fourth branch connected to L_c: there would be only four leakage inductances, whereas there are six short-circuit conditions to satisfy ($L_{sc_{12}}, L_{sc_{13}}, L_{sc_{14}}, L_{sc_{23}}, L_{sc_{24}}, L_{sc_{34}}$).

Similarly, for the integral flux approach, let us add a third cylindrical winding to the transformer. The equivalent magnetic circuit of Fig. 2(b) is shown in Fig. 4(b) and its dual electric circuit is shown in Fig. 5(b). Overall, for an N-winding transformer (or N-coil, if the windings are subdivided), there are $N(N-1)/2$ different short-circuit pairs. Hence, for a three-winding transformer, there are three short-circuit pairs: $L_{sc_{12}}, L_{sc_{13}}$ and $L_{sc_{23}}$. However, as can be observed in Fig. 5(b), there are only two leakage inductances and three short-circuit conditions to satisfy. For this reason, this approach is not valid for more than two windings.

The problem of finding an equivalent circuit for transformers with more than three windings is studied in [29, pp. 112–124]. For transformers with four windings, a star equivalent circuit is proposed in [33], as shown in Fig. 9, where the number of independent inductances is six (and the number of short-circuit conditions to satisfy is six, as mentioned earlier). The star equivalent circuit for the five-winding transformer is presented in [34] and is generalized for the N-winding transformer in [35]. However, these models are not topological, since each inductance of the star equivalent circuit has no physical meaning. Hence, in that case, the indefinite admittance matrix method (BCTRAN) should be used instead, because of its simplicity.

A similar but simpler approach is to remove some of the inductances in the star equivalent circuit (for instance, short-circuit L_{l_e} or L_{l_f} from Fig. 9) to build a *simplified star equivalent circuit*, as illustrated in Fig. 10. This name comes from the fact that the model has only $2N-3$ inductances, whereas the number of different short-circuit pairs is $N(N-1)/2$. It is shown in [25] that some equations are not independent and must be removed, in order to calculate all the parameters. This type of model is presented for instance in [36, Fig. 4], [2, Fig. 6.2], [37, Fig. 6], and [13, Fig. 4.10]. It can also be explained by adding the inductances L_{l_2} to $L_{l_{N-1}}$ in Fig. 10 to the integral flux approach to match short-circuit measurements. It is said that these inductances compensate for the finite radial thickness of windings 2 to $N-1$ [37].

It is proposed in [11] to use coupled leakage inductances for the three-winding transformer case. The motivation of [11] was to find an alternative to the star circuit, where there exists (for typical transformer designs) a negative leakage inductance that can lead to numerical instabilities during simulations. This method was generalized as a leakage model in [19], for a N-coil transformer, and is presented next. The addition of mutual leakage inductances in the integral flux approach allows to match short-circuit measurements.

The short-circuit inductance matrix $[L_{sc}]$ is of the following form:

$$[L_{sc}]_{N \times N} = \begin{bmatrix} 0 & L_{sc_{12}} & \cdots & L_{sc_{1N}} \\ L_{sc_{21}} & 0 & \cdots & L_{sc_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ L_{sc_{N1}} & L_{sc_{N2}} & \cdots & 0 \end{bmatrix} \quad (14)$$

where the diagonal elements are zero, since a short-circuit of a coil with itself is zero, and where the matrix is symmetric, since the short-circuit inductances are reciprocal.
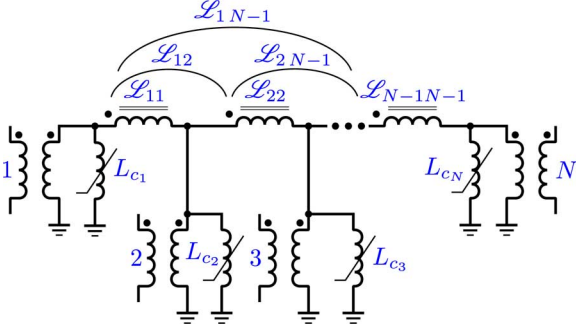
Fig. 11. Coupled leakage model for the single-phase N-winding shell-type transformer with cylindrical windings (reciprocal mutual inductances $\mathscr{L}_{ji}$ are not shown).

Fig. 12. Magnetic flux paths for the three-phase N-winding shell-type transformer with pancake windings.

From Figs. 3(b) and 5(b), it can be seen that for the integral flux approach, there are $N-1$ leakage inductances for an N-coil transformer and the dual electric circuit for the single-phase transformer can be generalized for N cylindrical coils to the one presented in Fig. 11. In matrix form, it is shown in [19] that the elements of this new coupled leakage inductance matrix $[\mathscr{L}]$ in terms of the short-circuit inductances are given by

$$\mathscr{L}(i,j) = \frac{1}{2}[L_{\mathrm{sc}}(i,j+1) + L_{\mathrm{sc}}(i+1,j)$$
$$- L_{\mathrm{sc}}(i,j) - L_{\mathrm{sc}}(i+1,j+1)]. \quad (15)$$

For the particular case of the three-winding transformer, the short-circuit inductance matrix $[L_{\mathrm{sc}}]$ is given by

$$[L_{\mathrm{sc}}] = \begin{bmatrix} 0 & L_{\mathrm{sc}_{12}} & L_{\mathrm{sc}_{13}} \\ L_{\mathrm{sc}_{12}} & 0 & L_{\mathrm{sc}_{23}} \\ L_{\mathrm{sc}_{13}} & L_{\mathrm{sc}_{23}} & 0 \end{bmatrix} \quad (16)$$

and the coupled leakage inductance matrix $[\mathscr{L}]$ is

$$[[\mathscr{L}] = \begin{bmatrix} L_{\mathrm{sc}_{12}} & \frac{(L_{\mathrm{sc}_{13}} - L_{\mathrm{sc}_{12}} - L_{\mathrm{sc}_{23}})}{2} \\ \frac{(L_{\mathrm{sc}_{13}} - L_{\mathrm{sc}_{12}} - L_{\mathrm{sc}_{23}})}{2} & L_{\mathrm{sc}_{23}} \end{bmatrix} \quad (17)$$

which is equivalent to [11, (7)–(8)]. Note that the conclusion in [11] that the mutual inductance is positive is not necessarily true if the windings have different heights. This is the case for the two transformers presented next, for which some of the mutual inductances are negative.

It should be emphasized here that this method is different from the one presented in [38], which was also termed the coupled leakage model. Indeed, by looking at its constitutive equation in [38, (4)], it can be seen that this is the classic self/mutual inductances representation of the transformer (as presented for instance in [14, (4)] and that it includes not only leakage inductances, but also core inductances.

V. METHODS

As a verification for the analytical method presented in [18] for the calculation of the short-circuit inductance matrix and also to test the new coupled leakage approach proposed by [19], short-circuits tests were performed on a single-phase, 360 MVA, $400/\sqrt{3}$ kV/24 kV, 50 Hz, two-winding shell-type transformer with pancake coils and a three-phase 96 MVA, 400 kV/3 × 6.8
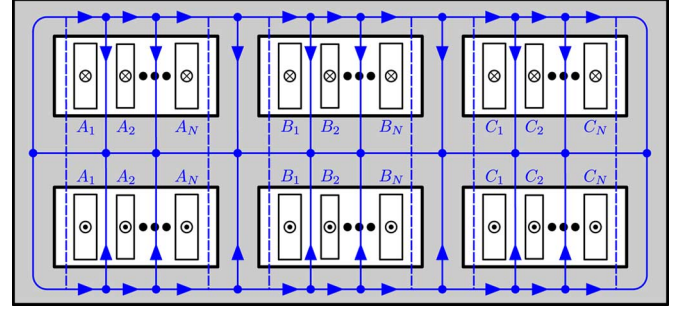
kV, 50 Hz, four-winding shell-type transformer with pancake coils. The windings of the 360 MVA transformer are divided into 44 pancake coils, and the four windings of the 96 MVA transformer are divided into 26 pancake coils per phase.

For single-phase shell-type transformers with pancake windings, it can be demonstrated that the dual electric circuit is the one shown in Fig. 11. For the three-phase shell-type transformer with pancake windings shown in Fig. 12, the magnetic equivalent circuit is illustrated in Fig. 13,[7] and its dual electric circuit is shown in Fig. 14.

Since the short-circuit measurements between coils are never available, the method presented in [18] was used to calculate the short-circuit inductance between each coil pair to fill up the short-circuit inductance matrix (14). With the short-circuit inductances known, the coupled leakage matrix (15) can be computed. The indefinite admittance matrix method presented in [14] (BCTRAN) was used to verify the results obtained with the approach presented in this paper. The commonly used short-circuit model of [14] is also used in hybrid topological models such as [12] and [4], with modifications to account for the fictitious windings. The same short-circuit inductance matrix $[L_{\mathrm{sc}}]$ was used for both methods.

Simulations were performed for both transformers with the two approaches and the short-circuit inductances between windings (coils are regrouped into windings) were calculated and compared with measurements. For the 96 MVA transformer, the positive-sequence short-circuit inductance was measured between the high-voltage winding (winding 1) and each of the low-voltage windings (2 to 4). In all simulation cases, the low-voltage winding is short-circuited and the high-voltage winding is energized with voltages corresponding to the measurements.

VI. RESULTS

The results for the short-circuit inductance of the 360 MVA transformer are presented in Table I between the HV and the LV windings. The difference between measurement and simulation is 0.04%.

As for the 96 MVA transformer, the positive-sequence short-circuit inductances are shown in Tables II–IV. Each low-voltage

[7]Note that in general, the magnetomotive forces of the center phase are reversed with respect to the outer phases. This is accounted for with the appropriate electrical connections to the sources (reversed source polarity for the center phase), outside the magnetic equivalent circuit.
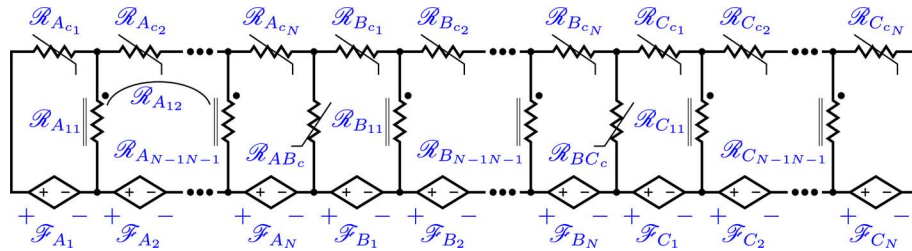
Fig. 13.   Equivalent magnetic circuit for the three-phase N-winding shell-type transformer with sandwiched windings (mutual reluctances $\mathscr{R}_{ij}$ are not shown, except for $\mathscr{R}_{A_{12}}$).
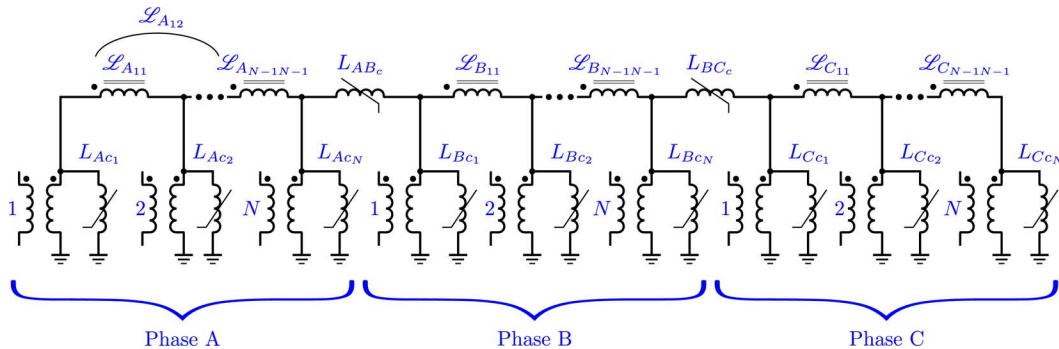

Fig. 14.   Coupled leakage model for the three-phase N-winding shell-type transformer with sandwiched windings (mutual inductances $\mathscr{L}_{ij}$ are not shown, except for $\mathscr{L}_{A_{12}}$).

TABLE I
SHORT-CIRCUIT INDUCTANCE FOR THE 360 MVA TRANSFORMER SEEN FROM THE HV SIDE

Method	Short-circuit inductance [mH]
Measurement	69.43
Coupled leakage model	69.4598
BCTRAN	69.4598

TABLE II
POSITIVE-SEQUENCE SHORT-CIRCUIT INDUCTANCE WINDINGS 1 AND 2 FOR THE 96 MVA TRANSFORMER SEEN FROM THE HV SIDE

Method	Short-circuit inductance [H]
Measurement	1.22
Coupled leakage model	1.2934
BCTRAN	1.2934

TABLE III
POSITIVE-SEQUENCE SHORT-CIRCUIT INDUCTANCE WINDINGS 1 AND 3 FOR THE 96 MVA TRANSFORMER SEEN FROM THE HV SIDE

Method	Short-circuit inductance [H]
Measurement	1.23
Coupled leakage model	1.2707
BCTRAN	1.2707

TABLE IV
POSITIVE-SEQUENCE SHORT-CIRCUIT INDUCTANCE WINDINGS 1 AND 4 FOR THE 96 MVA TRANSFORMER SEEN FROM THE HV SIDE

Method	Short-circuit inductance [H]
Measurement	1.27
Coupled leakage model	1.3707
BCTRAN	1.3707

VII.   DISCUSSION

It can be seen in Tables I–IV that there is no difference between the results of the coupled leakage method and the indefinite admittance matrix method (BCTRAN). Hence, from a short-circuit point of view, the two methods are equivalent. However, the coupled leakage model is topologically correct and can accommodate a topological core model without the use of fictitious windings, whereas one or more fictitious windings are necessary to connect the indefinite admittance matrix to the topological core in [4] and [12]. The establishment of such fictitious winding is arbitrary and is not supported by any physical basis. As such, the coupled leakage model proposed in [19], and generalized here for the three-phase shell-type transformer, appears to be a better and more intuitive approach for modeling leakage flux in topological transformer models.

Another interesting result observed from Tables I–IV is that the method presented in [18] for the calculation of the short-circuit inductance matrix $[L_{sc}]$, was validated with experimental short-circuit measurements for two transformers. The higher differences observed in the three-phase case can be explained by the fact that the 2D approximation used in the analytical method [18] used for evaluating the short-circuit inductances is less accurate for that case. Nevertheless, the results obtained are acceptable.

winding is tested separately. The errors for the short-circuit inductances of each low-voltage winding are 6.02%, 3.31% and 7.93%, respectively. Again, the error observed with BCTRAN comes from the fact that the method presented in [18] was used to calculate the short-circuit inductance between each coil pair to fill up the short-circuit inductance matrix, since short-circuit measurements between coils are never available.

VIII. CONCLUSION

In this paper, the differences between topological transformer models were explained through the concepts of *divided* and *integral* fluxes. It was shown that the divided flux approach is the result of mathematical manipulations and that the integral flux approach should be preferred, since it represents more closely the physical path of the flux lines in a transformer. Furthermore, a relationship was derived to calculate the split ratio of leakage inductances of the T-network model for the single-phase two-winding transformer, in the case where the core inductances are assumed linear. When the core inductances are nonlinear, it was demonstrated that nonlinear leakage inductances are necessary in the divided flux approach, in order for the models to be mathematically equivalent, whereas leakage inductances in the integral flux approach are linear by definition (flux tubes in air).

Also, a new model was proposed for the three-phase shell-type transformer that uses the coupled leakage model presented in [19]. This new approach has the advantage of modeling all short-circuit conditions and works for transformers with more than two windings (or coils), whereas the uncoupled leakage inductances in topological models are limited to a very specific number of windings (two for the integral flux model and three for the divided flux model).

Furthermore, it was shown that the indefinite admittance matrix (BCTRAN) and the coupled leakage model give the same short-circuit results. However, since the coupled leakage model is topologically correct, no fictitious winding is required, and the coupled inductances can be connected directly to the topological core model. This represents an important improvement over existing hybrid topological models.

Finally, the analytical method presented in [18] to calculate the short-circuit inductance matrix was verified for two transformers, and the results show good agreement between the measurements and the calculated short-circuit inductances.

APPENDIX

A numerical example for the mathematical equivalence of T and Π models is provided here. It is assumed that $l_{c_1} = l_{c_2}$, but it is not mandatory.

- Π-equivalent: $L_A = 2$ H, $L_B = 2$mH, $\lambda_{0_B} = 0.5994$ Wb, $L_{l_{12}} = 0.7$ mH.
- T-equivalent: $L_D = 0.3499$ mH, $L_E = 0.6991$ μH, $L_F = 0.2979$ mH, $\lambda_{0_E} = 0.2095$ mWb, $L_G = 0.9998$ H, $L_H = 1.9973$ mH, $L_I = 0.8511$ mH, $\lambda_{0_H} = 0.5986$ Wb, $\lambda_{0_I} = 0.5994$ Wb.

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Mathieu Lambert (S'08–M'09) received the B.Eng. and M.Sc. degrees in electrical engineering from the École Polytechnique de Montréal, Montréal, QC, Canada, in 2007 and 2009, respectively, where he is currently pursuing the Ph.D. degree in electrical engineering.

Manuel Martínez-Duró (M'09) received the M.Eng. degree in industrial engineering from the Universitat Politècnica de Catalunya, Barcelona, Spain, in 1999.

He joined Électricité de France, R&D Division, Clamart, France, in 2001, where he is a Researcher in power system transients.

Jean Mahseredjian (F'13) received the Ph.D. degree in electrical engineering from École Polytechnique de Montréal, Montréal, QC, Canada, in 1991.

From 1987 to 2004, he was with IREQ (Hydro-Québec), working on research-and-development activities related to the simulation and analysis of electromagnetic transients. In 2004, he joined the faculty of electrical engineering, École Polytechnique de Montréal.

Francisco de León (S'86–M'92–SM'02) received the B.Sc. and M.Sc. (Hons.) degrees in electrical engineering from the National Polytechnic Institute, Mexico City, Mexico, in 1983 and 1986, respectively, and the Ph.D. degree in electrical engineering from the University of Toronto, Toronto, ON, Canada, in 1992.

He has held several academic positions in Mexico and has worked for the Canadian electric industry. Currently, he is an Associate Professor at Polytechnic Institute of New York University (NYU), Brooklyn, NY, USA. His research interests include the analysis of power phenomena under nonsinusoidal conditions, the transient and steady-state analyses of power systems, the thermal rating of cables, and the calculation of electromagnetic fields applied to machine design and modeling.

Frédéric Sirois (S'96–M'05–SM'07) received the B.Eng. degree in electrical engineering from Université de Sherbrooke, Sherbrooke, QC, Canada, in 1997, and the Ph.D. degree in electrical engineering from École Polytechnique de Montréal, Montréal, QC, Canada, in 2003.

From 1998 to 2002, he was affiliated as a Ph.D. scholar with Hydro-Québec's Research Institute (IREQ). From 2003 to 2005, he was a Research Engineer at IREQ. In 2005, he joined École Polytechnique de Montréal where he is now Associate Professor. His main research interests include the modeling and design of electromagnetic and superconducting devices, integration studies of superconducting equipment in power systems, and planning of power systems. He is also a regular reviewer for several international journals and conferences.